06-362: Chemical Engineering Process Control

Filter Equations

Bruno Abreu Calfa

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This handout presents the corrected equations for a *first-order-low-pass filter* discussed in Chapter 5 of the notes. More information on Butterworth filters, which include the one we saw in class, can be found here (see the "Transfer Function" section) and a discussion about angles measured in radians and degrees having "non-unit" units can be found here.

To summarize, the amplitude ratio, AR, and phase shift, ϕ, are given by:

$$AR(\omega) = \frac{1}{\sqrt{\left(\frac{\omega}{\omega_B}\right)^2 + 1}} \tag{1}$$

$$\phi(\omega) = \arctan\left(-\frac{\omega}{\omega_B}\right) \tag{2}$$

where $\omega_B = \frac{1}{\tau}$ is the breakpoint/cutoff frequency and $\tau = \frac{1}{K_c}$ is the time constant (expressed in seconds, for example) and K_c is the controller gain.

Hence, from the definition of $AR(\omega_B) = \frac{1}{\sqrt{2}}$, it follows that:

$$AR(\omega_B) = \frac{1}{\sqrt{\left(\frac{\omega_B}{K_c}\right)^2 + 1}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\left(\frac{\omega_B}{K_c}\right)^2 + 1}}$$

$$\omega_B = K_c \tag{3}$$

The apparent inconsistency with the units is explained in the second link above. We should name the units for angles; however, in the end of the day they can be ignored in the dimensional analysis of an equation.