1 Compound Interest

$$S = P(1 + i/m)^{mn} \tag{1}$$

where:

n : compounding periods (years)
m : number of times the annual interest rate, i, is compounded per year
P : present value
S : lump sum or future worth

2 Annuity

End-Of-Year (EOY) payments:

$$X = P\left[\frac{i/m}{1 - (1 + i/m)^{-mn}}\right] \tag{2}$$

Beginning-Of-Year (BOY) payments:

$$X = P\left[\frac{i/m}{(1 + i/m) - (1 + i/m)^{-mn}}\right] \tag{3}$$

where:

X : payment/expense on a regular basis, also denoted R

3 Perpetuity ("Infinite" Time Period)

$$NPV = C_0 + \frac{CF}{i} + \frac{C}{(1 + i)^z - 1} \tag{4}$$

where:

NPV : Net Present Value
C_0 : original cost
CF : amount of cash flow
C : replacement cost, $C = C_0 - C_S$, where C_S is the salvage value
z : operating life

4 Discounted Cash Flow (DCF)

$$NPV = -\left(C_I + C_W\right) + \sum_{j=1}^{n} (R_j - X_j)(1-t)\frac{1}{(1+i)^j} + \sum_{j=1}^{n_t} D_j t \frac{1}{(1+i)^j} + (C_S + C_W)\frac{1}{(1+i)^n} \tag{5}$$

where:

C_I : fixed investment (initial unit cost)
C_W : working capital
n : total useful/project life
R_j : revenues in period j
X_j : expenses in period j
t : tax rate
i : interest rate
n_t : tax life
D_j : depreciation in period j
C_S : salvage value

For straight-line depreciation, $D_j = D$ (constant):

$$D = \frac{C_I - C_S}{n_t}$$

For uniform/constant R_j, X_j and D_j, the sums of the discount factors, $\frac{1}{(1+i)^j}$, simplify to:

$$NPV = -\left(C_I + C_W\right) + (R - X)(1-t)\left[\frac{1-(1+i)^{-n}}{i}\right] + Dt\left[\frac{1-(1+i)^{-n_t}}{i}\right] + (C_S + C_W)\frac{1}{(1+i)^n} \tag{6}$$

5 Cost Comparison for Different Project Lives

Three alternatives:

1. Assume perpetuity for each project and calculate NPVs
2. Convert projects lives to the same basis, using least common multiple (LCM) and calculate NPVs
3. Normalize all income and costs (NPVs) to an annualized basis

Alternative 3 yields:

$$\overline{NPV} = NPV \frac{i}{1-(1+i)^{-z}} \tag{7}$$

where:

$$\begin{aligned} \overline{NPV}: &\quad \text{normalized NPV} \\ z: &\quad \text{operating life} \end{aligned}$$

References

[1] L. T. Biegler, I. E. Grossmann, and A. W. Westerberg. *Systematic Methods of Chemical Process Design.* Prentice Hall PTR., Upper Saddle River, NJ, USA, 1997.