MS Excel and VBA

Module 2: Solver Tool

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Outline

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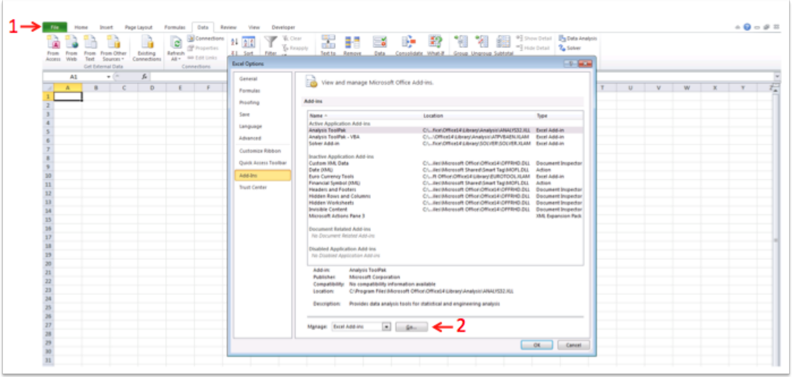
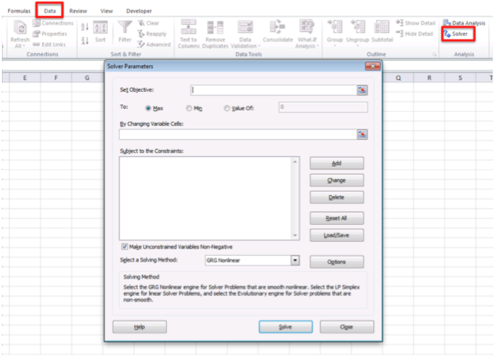
- It solves complex linear and nonlinear problems and can also be used in conjunction with VBA to automate tasks
- To enable the Solver add-in, go to `File` → `Options` → `Add-Ins` → `Go...` and make sure the option "Solver Add-in" is selected



Solver Parameters



- **Objective Cell:** cell which will represent the objective or goal

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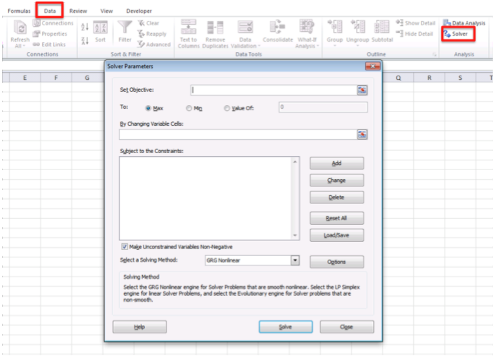

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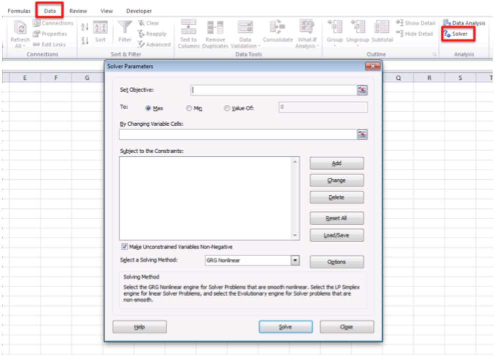

- **Objective Cell:** cell which will represent the objective or goal
- **Changing Cells:** cells that can change or adjust to optimize the target cell
- **Constraints:** restrictions/limitations that you apply on the changing cells

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Motivating Example

- The goal of optimization is to *maximize* or *minimize* an objective by systematically changing variables

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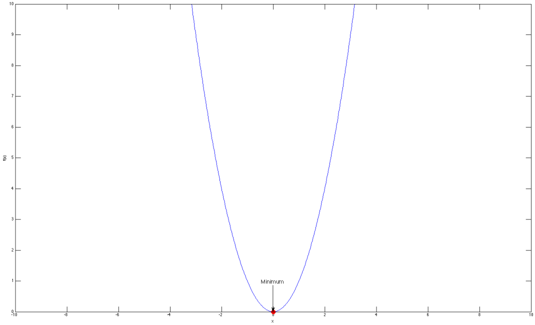

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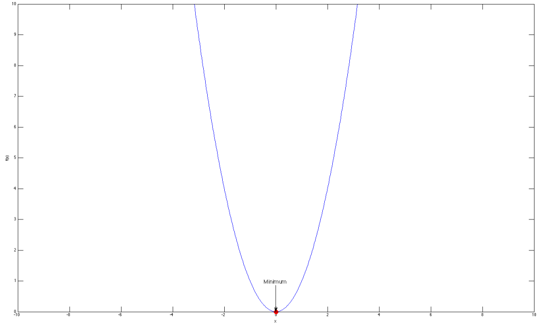


- Solution: $x^* = 0$ (minimizer) with $f(x^*) = 0$

Classes of Problems I

- Unconstrained Optimization
  - General formulation:

$$\boxed{\min_{x} \quad f(x)}$$

where:

x	Decision variable
$f(x)$	Objective function

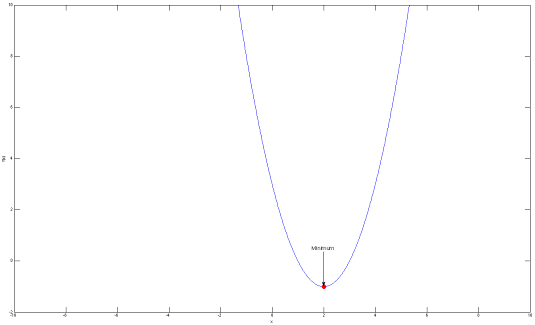
  - Example:

$$\min_{x} \quad x^2 - 4x + 3$$

  - Feasible region: all real values of x
  - **Remark:** $\boxed{\min f(x)}$ is equivalent to $\boxed{-[\max -f(x)]}$ (think of the parabola $f(x) = x^2 - 1$)

Classes of Problems II

- Plot of $f(x) = x^2 - 4x + 3$



- Solution: $x^* = 2$ with $f(x^*) = -1$

Classes of Problems III

- Constrained Optimization
  - General formulation:

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{s.t.} \quad & h(x) = 0 \\ & g(x) \leq 0 \end{aligned}$$

where:

x	Decision variable
$f(x)$	Objective function
$h(x)$	Equality constraints
$g(x)$	Inequality constraints

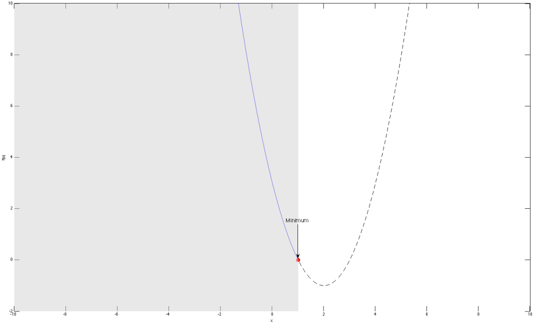
  - Example:

$$\begin{aligned} \min_{x} \quad & x^2 - 4x + 3 \\ \text{s.t.} \quad & x - 1 \leq 0 \end{aligned}$$

  - Feasible region: $x \in (-\infty,\ 1]$

Classes of Problems IV

- Plot of $f(x) = x^2 - 4x + 3$ and constraint $x \leq 1$



- Solution: $x^* = 1$ with $f(x^*) = 0$

Outline

Definition

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

where:

c	Cost coefficients
x	Decision variables
A	Matrix of coefficients of x
b	Vector of right-hand side elements

Example Problem I

- A refinery has available two crude oils that have the yields shown in the following table. Because of equipment and storage limitations, production of gasoline, kerosene, and fuel oil must be limited as also shown in this table. There are no plant limitations on the production of other products such as gas oils. The profit on processing crude #1 is $1.00/bbl and on crude #2 it is $0.70/bbl. Find the optimum daily feed rates of the two crudes to this plant.

	Volume percent yields		**Maximum allowable product rate bbl/day**
	Crude #1	**Crude #2**	
Gasoline	70	31	6,000
Kerosene	6	9	5,400
Fuel oil	24	60	5,000

Example Problem II

- Formulation:

$$\begin{aligned} \max_{x} \quad & x_1 + 0.7x_2 \\ \text{s.t.} \quad & 70x_1 + 31x_2 \leq 6000 \quad & \text{(Gasoline)} \\ & 6x_1 + 9x_2 \leq 5400 \quad & \text{(Kerosene)} \\ & 24x_1 + 60x_2 \leq 5000 \quad & \text{(Fuel oil)} \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

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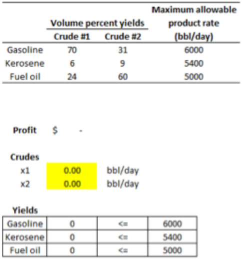
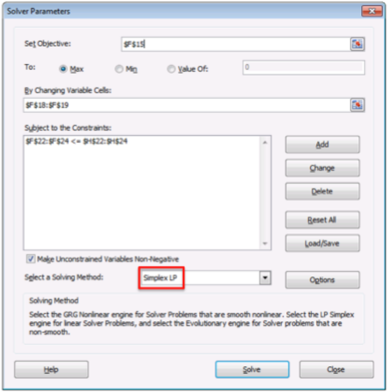


	Volume percent yields		Maximum allowable product rate
	Crude #1	Crude #2	(bbl/day)
Gasoline	70	31	6000
Kerosene	6	9	5400
Fuel oil	24	60	5000

Profit $ -

Crudes		
x1	0.00	bbl/day
x2	0.00	bbl/day

Yields			
Gasoline	0	<=	6000
Kerosene	0	<=	5400
Fuel oil	0	<=	5000



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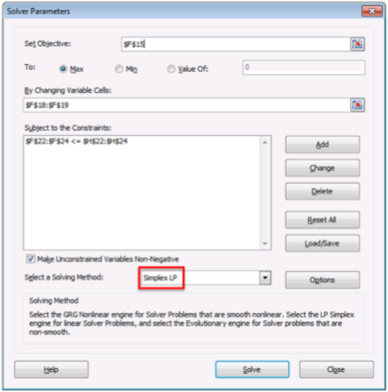

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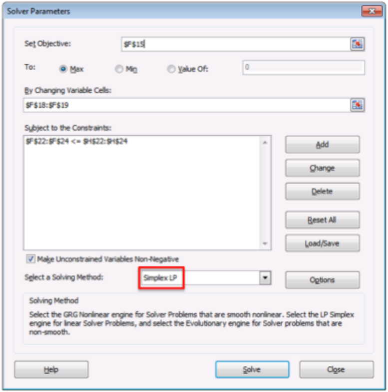

- Hit “Solve”
- See file **`Solver_Examples.xlsx`**, worksheet “LO Example”

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Definition

$$
\begin{aligned}
\min_{x} \quad & f(x) \\
\text{s.t.} \quad & h(x) = 0 \\
& g(x) \leq 0
\end{aligned}
$$

where:

x	Decision variable
$f(x)$	Objective function
$h(x)$	Equality constraints
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- At least one of the functions $f(\cdot)$, $g(\cdot)$, or $h(\cdot)$ is nonlinear

Example Problem

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$$Ax - b = 0$$

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- Idea: solve the following optimization problem

$$\begin{aligned} \min_x \quad & z \\ \text{s.t.} \quad & A_i x - b_i = 0 \qquad\qquad i = 1, \ldots, n \\ & z - \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} x_j - b_i) = 0 \end{aligned}$$

that is, *set* the sum of the *residues* to zero (in Solver, this is equivalent to making the objective cell as the sum of the residues and checking the option "Value Of:" with the value 0)

Example Problem: Definition

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- Idea: solve the following optimization problem

$$\min_{x} \quad \sum_{i=1}^{n} f_i^2(x)$$
$$\text{s.t.} \quad f_i(x) = 0 \quad i = 1, \ldots, n$$

that is, minimize the sum of the squares of the *residues*

Example Problem: Definition

- Chemical equilibrium of the combustion of propane (C_3H_8) and air (O_2 and N_2) to form ten products

$$C_3H_8 + \frac{R}{2}(O_2 + 4\,N_2) \longrightarrow \text{Products}$$

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- Taken from: Meintjes, K., Morgan, A. P. (1990) Chemical Equilibrium Systems as Numerical Test Problems. ACM Transactions on Mathematical Software. 16(2): 143-151.

Example Problem: Equations

$$
\begin{aligned}
f_1(n) &= n_1 + n_4 - 3 = 0 \\
f_2(n) &= 2n_1 + n_2 + n_4 + n_7 + n_8 + n_9 + 2n_{10} - R = 0 \\
f_3(n) &= 2n_2 + 2n_5 + n_6 + n_7 - 8 = 0 \\
f_4(n) &= 2n_3 + n_9 - 4R = 0 \\
f_5(n) &= K_5 n_2 n_4 - n_1 n_5 = 0 \\
f_6(n) &= K_6 n_2^{0.5} n_4^{0.5} - n_1^{0.5} n_6 \left(\frac{p}{n_T}\right)^{0.5} = 0 \\
f_7(n) &= K_7 n_1^{0.5} n_2^{0.5} - n_4^{0.5} n_7 \left(\frac{p}{n_T}\right)^{0.5} = 0 \\
f_8(n) &= K_8 n_1 - n_4 n_8 \left(\frac{p}{n_T}\right) = 0 \\
f_9(n) &= K_9 n_1 n_3^{0.5} - n_4 n_9 \left(\frac{p}{n_T}\right)^{0.5} = 0 \\
f_{10}(n) &= K_{10} n_1^2 - n_4^2 n_{10} \left(\frac{p}{n_T}\right) = 0
\end{aligned}
$$

where $n_T = \sum_{i=1}^{10} n_i$

- See file **`Solver_Examples.xlsx`**, worksheet “NLE Example”