MS Excel and VBA: Module 2

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1 Nonlinear Optimization Example

1.1 Cost of Constructing a Distillation Column

Adapted from: Edgar, T. F., Himmelblau, D. M. (2001) *Optimization of Chemical Processes*. McGraw-Hill. 2^{nd} edition. 672p.

The cost of constructing a distillation column can be written as

$$C = C_p AN + C_s HAN + C_f + C_d + C_b + C_L + C_x$$

where

$$\begin{aligned}
C &= \text{Total cost, \$} \\
C_p &= \text{Cost per square foot of plate area, \$/ft}^2 \\
A &= \text{Column column cross-sectional area, ft}^2 \\
N &= \text{Number of plates} \\
N_{\text{min}} &= \text{Minimum number of plates} \\
C_s &= \text{Cost of shell, \$/ft}^3 \\
H &= \text{Distance between plates, ft} \\
C_f &= \text{Cost of feed pump, \$} \\
C_d &= \text{Cost of distillate pump, \$} \\
C_b &= \text{Cost of bottoms pump, \$} \\
C_L &= \text{Cost of reflux pump, \$} \\
C_x &= \text{Other fixed costs, \$}
\end{aligned}$$

The problem is to minimize the total cost, once product specifications and the throughput are fixed and the product and feed pumping costs are fixed; that is, C_f, C_d, C_L, and C_b are fixed. After selection of the material of construction, the costs are determined; that is, C_p, C_s, C_x are also fixed.

The process variables can be related through two empirical equations:

$$\frac{L}{D} = \left[\frac{1}{1-(N_{\text{min}}/N)}\right]\left(\frac{L}{D}\right)_{\text{min}}$$
$$A = K(L+D)$$

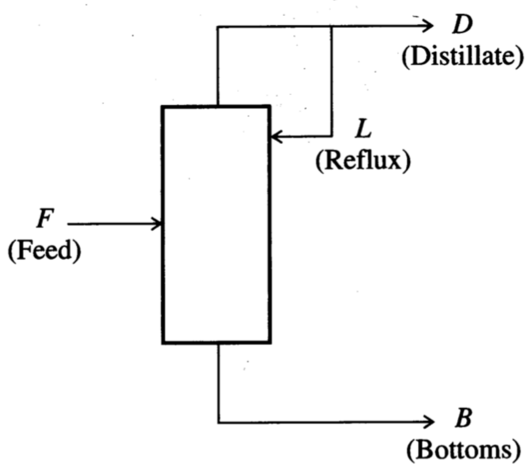

Figure 1: Distillation column schematic

For a certain separation and distillation column the following parameters are known to apply:

$$C_p = 30 \quad C_x = 8000 \quad C_s = 10 \quad F = 1500 \quad H = 2$$
$$D = 1000 \quad C_f = 4000 \quad C_f = 4000 \quad N_{\text{min}} = 5 \quad C_d = 3000$$
$$\left(\frac{L}{D}\right)_{\text{min}} = 1 \quad C_b = 2000 \quad K = 0.01$$

The pump cost for the reflux stream can be expressed as:

$$C_L = 5000 + 0.7L$$

Find the minimum total cost and corresponding values of the variables.

1.2 Solution

First we identify the decision (independent) variables: L, A, and N. Then we rewrite the constraints as follows:

$$L = \left[\frac{1}{1-(N_{\text{min}}/N)}\right]\left(\frac{L}{D}\right)_{\text{min}} D \tag{1}$$

$$A = K(L+D) \tag{2}$$

$$N \geq N_{\text{min}} \tag{3}$$

We want to minimize C subject to constraints (1), (2), (3). Obviously, we require that all decision variables are nonnegative. The number of stages is supposed to be an integer, but we will solve this problem by considering it to be a continuous variable for illustration purposes. Mixed-integer optimization problems are significantly harder to solve than purely continuous optimization problems. The initial guesses are: $L^{(0)} = 2000$, $A^{(0)} = 30\,\text{ft}^2$, and $N^{(0)} = 10$ stages.

Using the Solver add-in in MS Excel, the solution is:

$$C^* = 38,200\,\$$$

$$L^* = 2250$$

$$A^* = 32.50\,\text{ft}^2$$

$$N^* = 9 \text{ stages}$$

Notice that the success of Solver is strongly dependent on the initial guesses.