MS Excel and VBA

Module 2: Solver Tool

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Outline

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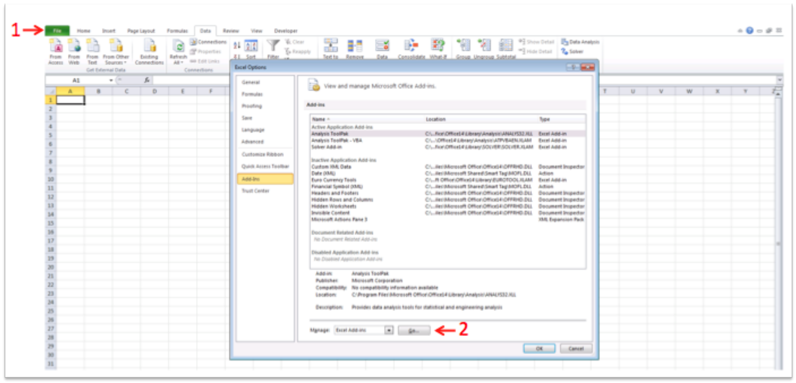
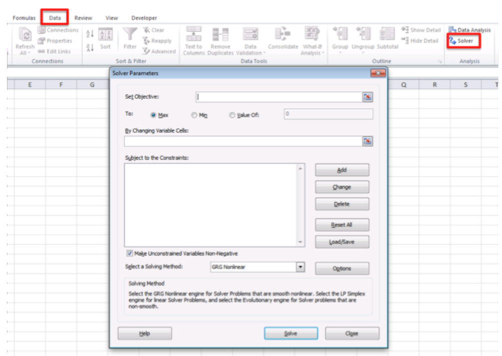
1 The Solver Tool

What is Solver?

- The Solver is an add-in for MS Excel, which is used for the optimization and simulation of business and engineering models
- It solves complex linear and nonlinear problems and can also be used in conjunction with VBA to automate tasks
- To enable the Solver add-in, go to `File` → `Options` → `Add-Ins` → `Go...` and make sure the option "Solver Add-in" is selected



Solver Parameters

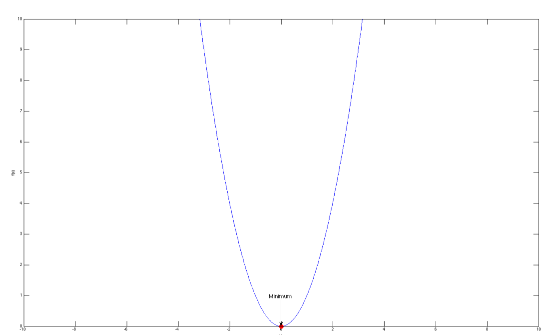


- **Objective Cell:** cell which will represent the objective or goal
- **Changing Cells:** cells that can change or adjust to optimize the target cell
- **Constraints:** restrictions/limitations that you apply on the changing cells

2 Optimization: Overview

Motivating Example

- The goal of optimization is to *maximize* or *minimize* an objective by systematically changing variables
- For example: find the minimum of the function $f(x) = x^2$ over all real values of x



- Solution: $x^* = 0$ (minimizer) with $f(x^*) = 0$

Classes of Problems

- Unconstrained Optimization
  - General formulation:

$$\boxed{\min_{x} \quad f(x)}$$

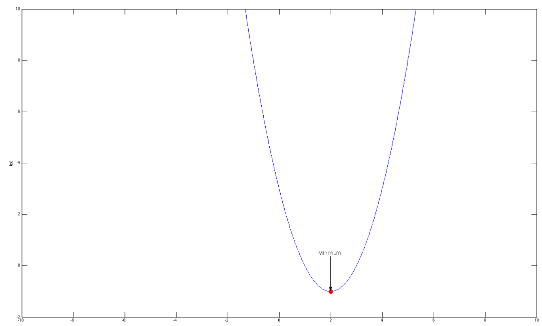
where:

x	Decision variable
$f(x)$	Objective function

  - Example:

$$\min_{x} \quad x^2 - 4x + 3$$

  - Feasible region: all real values of x
  - **Remark:** $\boxed{\min f(x)}$ is equivalent to $\boxed{-[\max -f(x)]}$ (think of the parabola $f(x) = x^2 - 1$)
  - Plot of $f(x) = x^2 - 4x + 3$



  - Solution: $x^* = 2$ with $f(x^*) = -1$

- Constrained Optimization
  - General formulation:

$$\boxed{\begin{array}{ll} \min_{x} & f(x) \\ \text{s.t.} & h(x) = 0 \\ & g(x) \leq 0 \end{array}}$$

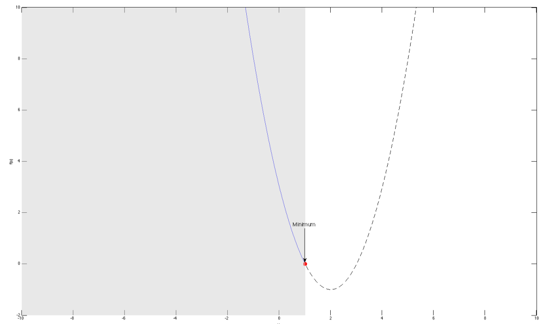
where:

x	Decision variable
$f(x)$	Objective function
$h(x)$	Equality constraints
$g(x)$	Inequality constraints

  - Example:

$$\begin{array}{ll} \min_{x} & x^2 - 4x + 3 \\ \text{s.t.} & x - 1 \leq 0 \end{array}$$

  - Feasible region: $x \in (-\infty,\ 1]$
  - Plot of $f(x) = x^2 - 4x + 3$ and constraint $x \leq 1$



- Solution: $x^* = 1$ with $f(x^*) = 0$

3 Linear Optimization

Definition

$$\begin{array}{ll} \min\limits_x & c^T x \\ \text{s.t.} & Ax \leq b \\ & x \geq 0 \end{array}$$

where:

c	Cost coefficients
x	Decision variables
A	Matrix of coefficients of x
b	Vector of right-hand side elements

Example Problem

- A refinery has available two crude oils that have the yields shown in the following table. Because of equipment and storage limitations, production of gasoline, kerosene, and fuel oil must be limited as also shown in this table. There are no plant limitations on the production of other products such as gas oils. The profit on processing crude #1 is $1.00/bbl and on crude #2 it is $0.70/bbl. Find the optimum daily feed rates of the two crudes to this plant.

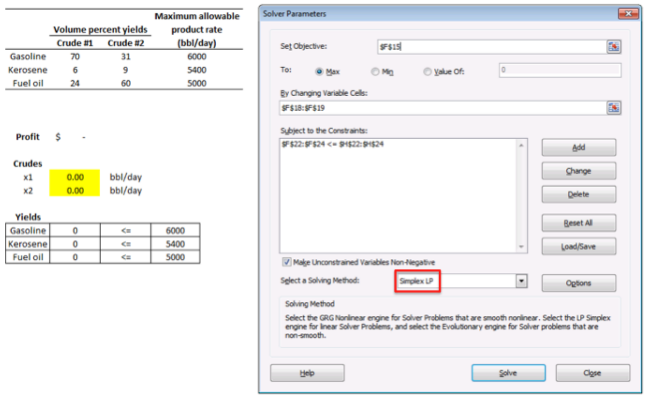
	Volume percent yields		**Maximum allowable product rate**
	Crude #1	**Crude #2**	**bbl/day**
Gasoline	70	31	6,000
Kerosene	6	9	5,400
Fuel oil	24	60	5,000

- Formulation:

$$\begin{array}{lll} \max\limits_x & x_1 + 0.7x_2 & \\ \text{s.t.} & 70x_1 + 31x_2 \leq 6000 & \text{(Gasoline)} \\ & 6x_1 + 9x_2 \leq 5400 & \text{(Kerosene)} \\ & 24x_1 + 60x_2 \leq 5000 & \text{(Fuel oil)} \\ & x_1,\ x_2 \geq 0 & \end{array}$$

Using Solver

- Enter the coefficients and formulas
- Open Solver and set its parameters as follows:



- Hit "Solve"
- See file **`Solver_Examples.xlsx`**, worksheet "LO Example"

4 Nonlinear Optimization

Definition

$$
\begin{aligned}
\min_{x} \quad & f(x) \\
\text{s.t.} \quad & h(x) = 0 \\
& g(x) \leq 0
\end{aligned}
$$

where:

x	Decision variable
$f(x)$	Objective function
$h(x)$	Equality constraints
$g(x)$	Inequality constraints

- At least one of the functions $f(\cdot)$, $g(\cdot)$, or $h(\cdot)$ is nonlinear

Example Problem

- Cost minimization of a distillation column
- See file **`NLO_Example.pdf`** for problem description
- See file **`Solver_Examples.xlsx`**, worksheet "NLO Example"

5 System of Linear Equations

Definition

- We want to solve the following:

$$Ax - b = 0$$

where

x	Vector of variables
A	Matrix of coefficients
b	Vector of right-hand-side elements

- Idea: solve the following optimization problem

$$\begin{aligned} \min_{x} \quad & z \\ \text{s.t.} \quad & A_i x - b_i = 0 \qquad\qquad i = 1,\ \ldots,\ n \\ & z - \sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij}x_j - b_i) = 0 \end{aligned}$$

that is, *set* the sum of the *residues* to zero (in Solver, this is equivalent to making the objective cell as the sum of the residues and checking the option "Value Of:" with the value 0)

Example Problem: Definition

- Linear Material Balances in a Process Flowsheet for ethanol production
- See file **`LE_Example.pdf`** for problem description
- See file **`Solver_Examples.xlsx`**, worksheet "LE Example"

6 System of Nonlinear Equations

Definition

- We want to solve the following:

$$f(x) = 0$$

where

x	Vector of variables
$f(x)$	Vector of functions (at least one is nonlinear)

- Idea: solve the following optimization problem

$$\begin{aligned} \min_{x} \quad & \sum_{i=1}^{n} f_i^2(x) \\ \text{s.t.} \quad & f_i(x) = 0 \quad i = 1,\ \ldots,\ n \end{aligned}$$

that is, minimize the sum of the squares of the *residues*

Example Problem: Definition

- Chemical equilibrium of the combustion of propane (C_3H_8) and air (O_2 and N_2) to form ten products

$$C_3H_8 + \frac{R}{2}(O_2 + 4\,N_2) \longrightarrow \text{Products}$$

- There are 10 variables and 10 nonlinear equations
- All parameters are given
- Taken from: Meintjes, K., Morgan, A. P. (1990) Chemical Equilibrium Systems as Numerical Test Problems. ACM Transactions on Mathematical Software. 16(2): 143-151.

Example Problem: Equations

$$f_1(n) = n_1 + n_4 - 3 = 0$$
$$f_2(n) = 2n_1 + n_2 + n_4 + n_7 + n_8 + n_9 + 2n_{10} - R = 0$$
$$f_3(n) = 2n_2 + 2n_5 + n_6 + n_7 - 8 = 0$$
$$f_4(n) = 2n_3 + n_9 - 4R = 0$$
$$f_5(n) = K_5 n_2 n_4 - n_1 n_5 = 0$$
$$f_6(n) = K_6 n_2^{0.5} n_4^{0.5} - n_1^{0.5} n_6 \left(\frac{p}{n_T}\right)^{0.5} = 0$$
$$f_7(n) = K_7 n_1^{0.5} n_2^{0.5} - n_4^{0.5} n_7 \left(\frac{p}{n_T}\right)^{0.5} = 0$$
$$f_8(n) = K_8 n_1 - n_4 n_8 \left(\frac{p}{n_T}\right) = 0$$
$$f_9(n) = K_9 n_1 n_3^{0.5} - n_4 n_9 \left(\frac{p}{n_T}\right)^{0.5} = 0$$
$$f_{10}(n) = K_{10} n_1^2 - n_4^2 n_{10} \left(\frac{p}{n_T}\right) = 0$$

where $n_T = \sum_{i=1}^{10} n_i$

- See file **`Solver_Examples.xlsx`**, worksheet “NLE Example”